Linear Algebra Applied to Kinematic Control of Mobile Manipulators

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Abstract. This paper is focused in linear algebra theory applied to control of mobile manipulator robots. In order to design the control algorithm, the kinematic system is approximated using numerical methods. Then, the optimal control actions are obtained through linear algebra approach. The structure of the controller consists in two solutions; a particular solution that allow following the desired trajectory and a homogeneous solution that allow performing secondary objectives as maximum manipulability and avoid static obstacles. In addition, the stability analysis is demonstrated through linear algebra concepts where it is shown that the tracking error tends asymptotically to zero. Finally, experimental results show the effective of proposed control algorithm over the mobile manipulator robot AKASHA.

Keywords: Linear algebra · Numerical methods · Controller design · Model · Mobile manipulator

1 Introduction

Robotics missions have evolved into the service domain where robots are expected to either exploit unknown dynamic environments, interact with human beings or manipulate dangerous products. Mobile manipulators combine these capabilities, thus expanding the working space, often characterized by a high degree of redundancy, combining the manipulation of a fixed base robotic arm with the mobility of a wheeled platform. Such systems allow the most common missions of robotic systems requiring locomotion and manipulation skills [1, 2]. Field or service robotics applications are numerous and all involve robots whose workspace capabilities have to be extended and whose control architecture and strategies must ensure a good overall performance in complex missions [3].

In the last decades, there has been a great deal of interest in mobile robots where, mobile manipulators being an area of great interest to researchers, who are looking for new non-linear control strategies. In [4] solves the trajectory-tracking problem by combining neural networks and robust control. The nonlinear mapping characteristic of neural networks and robust control are integrated in an adaptive control algorithm for mobile manipulator robots with non-linearities, perturbations and non-holonomic constraints all at the simulation level. PD feed-forward non-linear control is developed

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in [5], this control makes use of the knowledge of the mathematical model of the system and the measurement of the perturbations of process; is applied in a virtual prototype where the control parameters must be known for the controller to work well; [6] suggests a fuzzy PD controller to adjust parameters online depending on the state of the dynamic system. Other advanced control strategies are implemented, for example in [7] introduces a predictive control algorithm that has restrictions for a holonomic mobile manipulator robot; constraints such as acceleration, velocity, position, and avoiding obstacles are considered [8].

Control based on linear algebra is an innovative technique with main characteristics of the non-necessity of complex calculations to reach the control signal and the simplicity to make mathematical operations. [9–11]. In addition, the algorithm is easy to understand and implement, it allows the direct adaptation to any micro-controller without the need to make use of an external computer [12]. Due the fact that it is not a complex algorithm, this can be run on low processing power drivers, [13] by presenting a high yield on conventional computers, as a result, this algorithm supports savings in processing time and energy at the moment of executing the desired task [14–16]. In [17] presents a control algorithm based on linear algebra for mobile manipulator robots, but this work does not considered the null space configuration and the design is validated using simulation.

This paper proposes an algorithm of control based on linear algebra approach for trajectory tracking tasks of mobile robotic systems, formed by a robotic arm mounted on a mobile platform; the controller is based on kinematics and redundancy of the system. The structure of the control law consists of two solutions: (1) one particular solution that allow to meet the objective of the main task; and (2) a homogeneous solution that allow performing one or more secondary objectives, this work is considered the internal configuration of the mobile manipulator control in order to avoid singular configurations of the system and avoid static obstacles. The designed controller is validated experimentally. In addition, the stability is demonstrated through linear algebra concepts.

This article is organized into 4 Sections. Sect. 2 presents the modeling, design of the control algorithm and the analysis of stability based on approaches of linear algebra for the mobile manipulator. The discussion of experimental results is shown in Sect. 3, and the conclusions of the article in Sect. 4.

2 Modeling and Control Design

In this section, the kinematic model and control law based on linear algebra theory and numerical methods of the mobile manipulator is presented. In addition, the stability is obtained through linear algebra concepts.

2.1 Kinematic Model

The instantaneous kinematic model of a mobile manipulator gives the derivative of its end-effector location as a function of the derivatives of both the robotic arm configuration and the location of the mobile platform.

$$\dot{\mathbf{h}}(t) = \mathbf{J}(\mathbf{q})\mathbf{v}(t) \tag{1}$$

where $\dot{\mathbf{h}} = [\,\dot{h}_1 \quad \dot{h}_2 \quad \cdots \quad \dot{h}_m\,]^T$ is the vector of end-effector velocity, $\mathbf{v} = [\,v_1 \quad v_2 \quad \cdots \quad v_{\delta_n}\,]^T = [\,v_p^T \quad v_a^T\,]^T$ is the vector of mobile manipulator velocities in which contains the linear and angular velocities of the mobile platform and contains the joint velocities of robotic arm and $\mathbf{J}(\mathbf{q})$ is the Jacobian matrix that defines a linear mapping between the vector of the mobile manipulator velocities $\mathbf{v}(t)$and the vector of the end-effector velocity [17, 18].

2.2 Kinematic Controller

Considering the first order differential equation

$$\dot{h} = f(h, v, t)\,\text{con}\,h(0) = h_0$$

where, h represents the output of the system to be controller, $\dot{h}$ first derivative, v the control action and t the time. The values of $h(t)$ in the discrete time $t = k\,T_0$ are called $h(k)$ where, T_0 represents the sampling time and $k \in \{0, 1, 2, 3, 4, 5\ldots\}$. In addition, as mentioned in [17] the use of numerical methods for the calculus of the system evolution is based mainly on the possibility to approximation the state system at the instant time $k + 1$, if the state and the control action on the time instant k are known, this approximation is called Euler method.

$$h(k+1) = h(k) + T_0 f(h, u, t) \tag{2}$$

In order to design the controller, the model (1) can be approximated as (2)

$$\mathbf{h}(k+1) = \mathbf{h}(k) + T_0\mathbf{J}(\mathbf{q}(k))\mathbf{v}(k) \tag{3}$$

In addition, for that the tracking error tends to zero the following expression is used [17]

$$\mathbf{h}(k+1) = \mathbf{h_d}(k+1) - \mathbf{W}(\mathbf{h_d}(k) - \mathbf{h}(k)) \tag{4}$$

where, $\mathbf{h_d}$ is the desired trajectory $\mathbf{W}$ is a diagonal matrix and its values $0 <$ $\mathbf{diag}(w_{hx}, w_{hy}, w_{hz}) < 1$ are design parameters of the proposed controller.

Remark 1. If a faster response is required, the values should be closer to 0 and if a slower response is required, the values of should be closer to1.

Now, to generate the system equations consider (3) and (4) then, the system can be rewritten as $\mathbf{Au} = \mathbf{b}$.

$$\underbrace{\mathbf{J}(\,\mathbf{q}(k)\,)}_{\mathbf{A}}\underbrace{\mathbf{v}(k)}_{\mathbf{u}} = \underbrace{\frac{\mathbf{h_d}(k+1) - \mathbf{W}(\mathbf{h_d}(k) - \mathbf{h}(k)) - \mathbf{h}(k)}{T_0}}_{\mathbf{b}} \tag{5}$$

Note, that the Jacobian matrix has more unknowns that equation ($m < n$) therefore, an infinite solution exists to (5). A viable solution method is to formulate the problem as a constrained linear optimization problem.

$$\frac{1}{2}(\mathbf{v} - \mathbf{v_0})^T(\mathbf{v} - \mathbf{v_0}) = \min \tag{6}$$

Finally, a law of control for our system given by (5) is obtained minimizing (6):

$$\mathbf{v}_{ref} = \underbrace{\mathbf{J}^T\left(\mathbf{JJ}^T\right)^{-1}\mathbf{b}}_{\mathbf{v}_p} + \underbrace{\left(\mathbf{I}_n - \mathbf{J}^T\left(\mathbf{JJ}^T\right)^{-1}\mathbf{J}\right)\mathbf{v_0}}_{\mathbf{v}_h} \tag{7}$$

where the first term on the left-hand side is the particular solution ($\mathbf{v}_p$) and second term ($\mathbf{v}_h$) of this equation belong to the null space$\mathbf{J}$. In this work two different secondary objectives are considered: the obstacles avoid by the mobile platform and the singular configuration prevention through the systems manipulability control and is given by $\mathbf{v_0} = \mathbf{H}\left(\left[\,u_{obs} \quad \omega_{obs} \quad \left(\mathbf{q_d}^T - \mathbf{q^T}\right)\,\right]\right)$, where $\mathbf{H}$ is a diagonal matrix allows to increase, reduce or cancel the effect each objective, $(\mathbf{q_d}^T - \mathbf{q^T})$ are configuration errors of the mobile robotic arm, in such a way that the manipulator joints will be pulled to the desired values that maximize manipulability and u_{obs}, ω_{obs} are the linear velocity and angular velocity of the mobile platform respectively that avoids the static obstacles by resourcing to the null space configuration [18].

2.3 Stability Analysis

Considering the hypothesis of perfect velocity tracking, i.e., $\mathbf{v}_{\mathrm{ref}} = \mathbf{v}$, (7) can be substituted into the kinematic model (3) to obtain the following closed-loop equation:

$$\mathbf{h}(k+1) - \mathbf{h}(k) = T_0\mathbf{J}\left(\mathbf{J}^T(\mathbf{JJ}^T)^{-1}\mathbf{b} + \left(\mathbf{I}_n - \mathbf{J^T}\left(\mathbf{JJ^T}\right)^{-1}\mathbf{J}\right)\mathbf{v}_0\right) \tag{8}$$

where $\mathbf{JJ}^T(\mathbf{JJ}^T)^{-1} = \mathbf{I}_m$ the Eq. (6) is reduced

$$\mathbf{h}(k+1) - \mathbf{h}(k) = T_0\mathbf{I}_m\mathbf{b} + (\mathbf{JI}_n - \mathbf{I}_m\mathbf{J})\mathbf{v}_0 \tag{9}$$

through the properties of an identity matrix is achieved

$$\mathbf{h}(k+1) - \mathbf{h}(k) = T_0\left(\frac{\mathbf{h_d}(k+1) - \mathbf{W}(\mathbf{h_d}(k) - \mathbf{h}(k)) - \mathbf{h}(k)}{T_0}\right) \tag{10}$$

reducing terms and grouping is calculated the error in the following state $\mathbf{h_d}(k + 1) - \mathbf{h}(k + 1)$ depends only on the previous error for a gain $\mathbf{W}(\mathbf{h_d}(k) - \mathbf{h}(k))$ to,

$$\begin{bmatrix} e_{hx}(k+1) \\ e_{hy}(k+1) \\ e_{hz}(k+1) \end{bmatrix} = \begin{bmatrix} w_{hx}(e_{hx}(k)) \\ w_{hy}\left(e_{hy}(k)\right) \\ w_{hz}(e_{hz}(k)) \end{bmatrix}$$

The errors on the following states comes by

$$\begin{gathered} e_i(k+1) = w_i e_i(k) \\ e_i(k+2) = w_i e_i(k+1) = w_i^2 e_i(k) \\ e_i(k+3) = w_i e_i(k+2) = w_i^3 e_i(k) \\ \vdots \\ e_i(k+n) = w_i e_i(k+n-1) = w_i^n e_i(k) \end{gathered}$$

therefore the error tends asymptotically to zero when $0 < w_{\mathrm{i}} < 1$ and $n \to \infty$

3 Experimental Results

In this section, the performance of the proposed control law is tested throught the experimentation over the AKASHA robot which one consist of a robotic arm within 5DOF mount over a unicycle-type mobile platform. This platform allows linear velocity and angular velocity as a reference signal, (Fig. 1)

Fig. 1. AKASHA: Manipulator Mobile Robot

The experiment corresponds to the control system presents in (7). The desired trajectory for the end-effector of the mobile manipulator is described by $\mathbf{h_d} = [h_{xd} \quad h_{yd} \quad h_{zd}]^T$, where $h_{xd} = 0.5 + 0.2t$, $h_{yd} = 0.3\ \sin\ (0.4t)$ and $h_{zd} = 0.5 + 0.1\ \sin\ (0.4t)$. In this experiment, the mobile platform starts at $\mathbf{q}_p = [0\ \text{m} \quad 0\ \text{m} \quad 0\ \text{rad}]^T$; the robotic arm at $\mathbf{q}_a = \left[-\frac{29\pi}{36}\text{rad} \quad \frac{2\pi}{3}\ \text{rad} \quad \frac{4\pi}{9}\text{rad}\right]^T$ and $\mathbf{v_0} = \mathbf{H}([0 \quad 0 \quad (0\ \text{rad} - q_1)\ (1.22\ \text{rad} - q_2)(-1.05\ \text{rad} - q_3)])$.

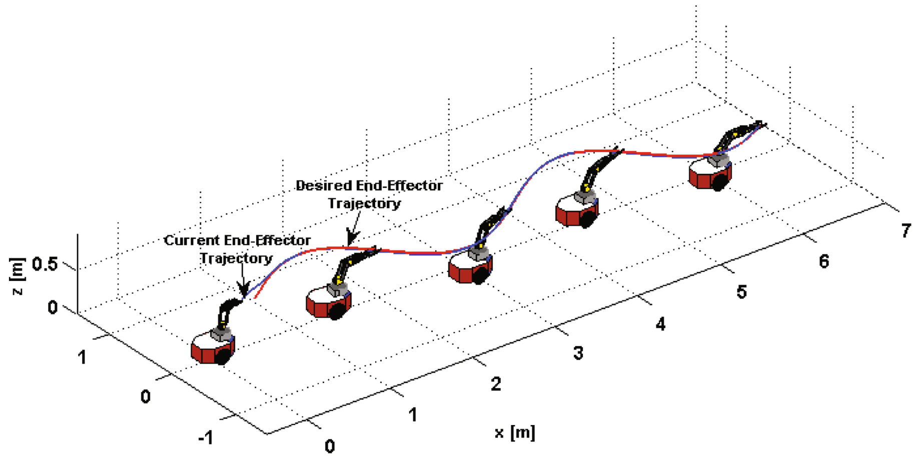

Fig. 2. Stroboscopic movement of the mobile manipulator in the trajectory tracking and internal configuration the of arm.

Figures 2, 3, 4 and 5 represent the experimental results. Figure 2, shows the desired trajectory and the current trajectory of the end-effector. It can be seen that the proposed controller presents a good performance while, considers the internal configuration of arm. Figure 3, shows the evolution of the tracking errors, which remain close to zero, while Figs. 4 and 5 show the optimal control actions.

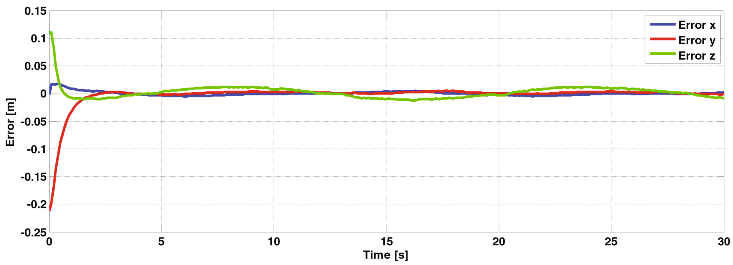

Fig. 3. Control errors of the mobile manipulator

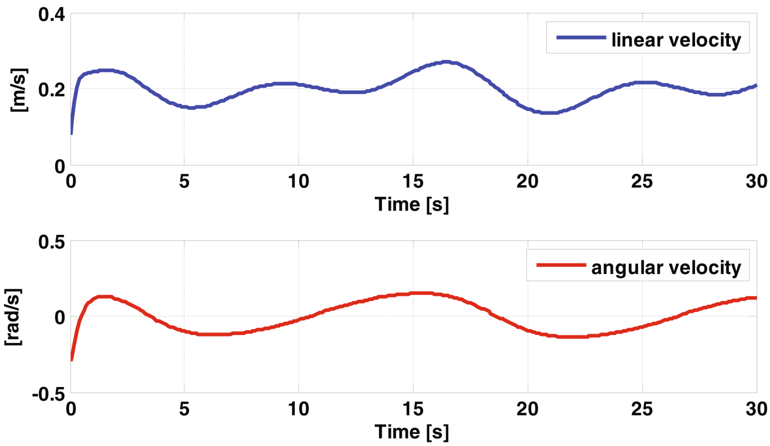

Fig. 4. Velocity commands to the mobile platform

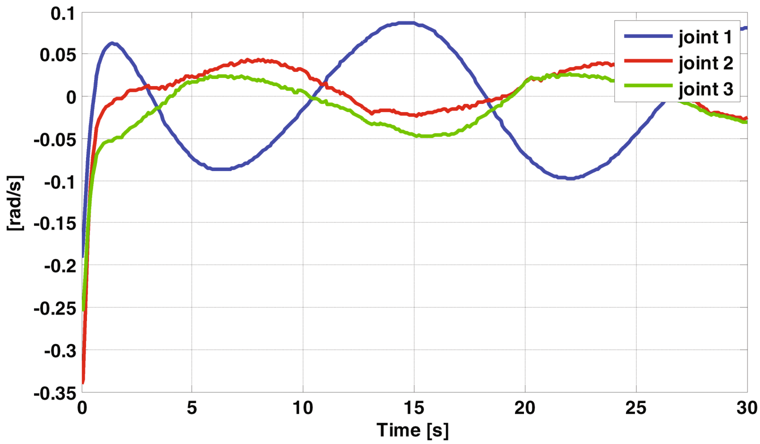

Fig. 5. Joint velocity commands to the robotic arm

For the second experiment, it's consider the avoidance of obstacles and maxima manipulability where, the environment is formed of a straight line and two static obstacles, $\mathbf{v_0} = \mathbf{H}\left(\left[u_{obs} \quad \omega_{obs} \quad \left(\mathbf{q_d}^T - \mathbf{q^T}\right)\right]\right)$ with $\mathbf{q_d} = [-0.698 \quad 2.27 \quad -2.27]$ to maintain maxima manipulability of the mobile manipulator during task execution, the mobile platform starts at $\mathbf{q_p} = [0 \text{ m} \quad -1 \text{ m} \quad 0 \text{ rad}]^T$ and robotic arm at $\mathbf{q_a} = [0 \text{ rad} \quad 0 \text{ rad} \quad 0 \text{ rad}]^T$.

Figures 6 and 7, shows the desired trajectory and the current trajectory of the end-effector. It can be seen that the proposed controller allow to meet the desired tracking trajectory while, avoiding static obstacles and considers the maxima manipulability. Figure 8, shows the evolution of the tracking errors, which remain close to zero.

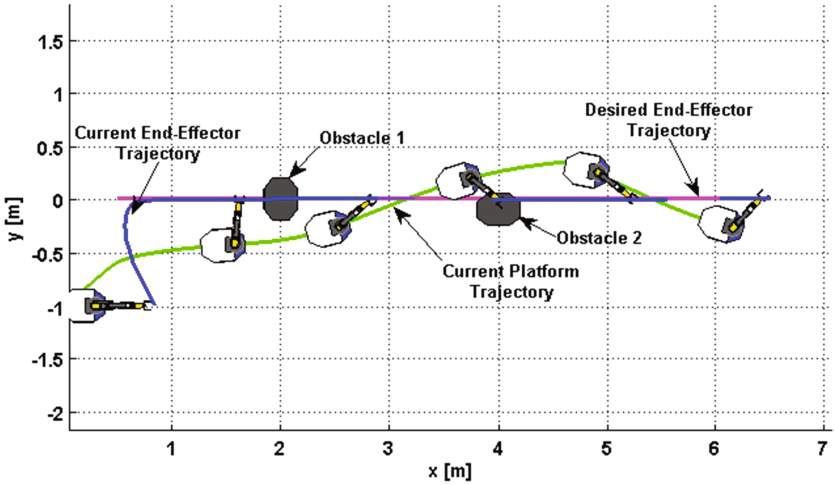

Fig. 6. Stroboscopic movement in two dimensions of the mobile manipulator in the trajectory tracking experiment with avoidance of obstacles and maxima manipulability

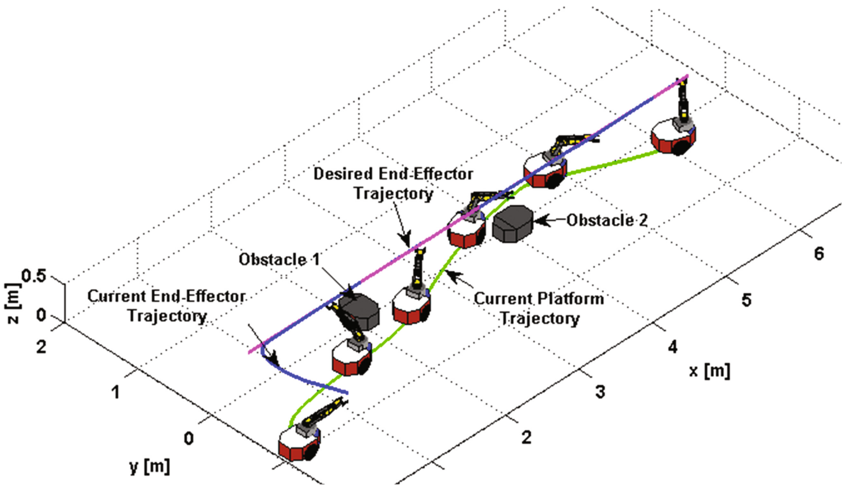

Fig. 7. Stroboscopic movement in three dimension of the mobile manipulator in the trajectory tracking experiment with avoidance of obstacles and maxima manipulability

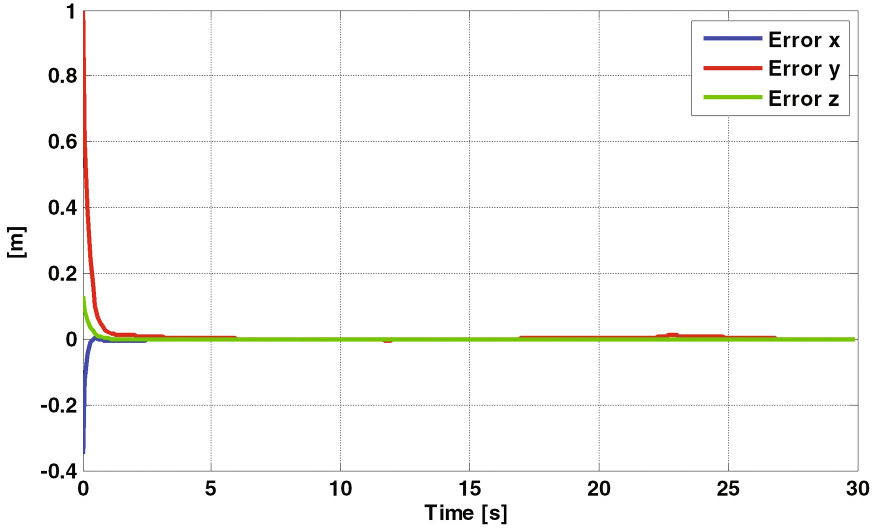

Fig. 8. Control errors of the mobile manipulator with avoid static obstacles

4 Conclusions

In this work a control algorithm based on linear algebra and numerical methods for mobile manipulator robots has been presented An advantage of this controller is its simple implementation in any programming language. The proposed control algorithm stability and performance has been analytically demonstrated trough the linear algebraic concepts. Two types of experiments were performed: one for follow desired trayectory while consider internal configuration of arm and another for follow desired trayectory while consider the avoidance of static obstacles and maxima manipulabilty. Experimental results shown the optimal performance of this control algorithm.

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References

1. Andaluz, V., Chicaiza, F., Gallardo, C., Quevedo, W., Varela, J., Sánchez, J., Arteaga, O.: Unity3D-MatLab simulator in real time for robotics applications. Lecture Notes in Computer Science, pp. 246–263. Springer, New York (2016)
2. Castellanos, E., García-Sánchez, C., Llanganate, W., Andaluz, V., Quevedo, W.: Robots coordinated control for service tasks in virtual reality environments. Lecture Notes in Computer Science, pp. 164–175. Springer, Cham (2017)

3. Andaluz, V., Quevedo, W., Chicaiza, F., Varela, J., Gallardo, C., Sánchez, J., Arteaga, O.: Transparency of a bilateral tele-operation scheme of a mobile manipulator robot. Lecture Notes in Computer Science, pp. 228–245. Springer, New York (2016)
4. Naijian, C., Hao, Y., Xiangdong, H., Changsheng, A., Chenglong, T., Xiangkui, L.: Adaptive robust control of the mobile manipulator based on neural network. In: 2016 IEEE 11th Conference on Industrial Electronics and Applications (ICIEA) (2016)
5. Meng, Z., Liang, X., Andersen, H., Ang, M.: Modelling and control of a 2-link mobile manipulator with virtual prototyping. In: 2016 13th International Conference on Ubiquitous Robots and Ambient Intelligence (URAI) (2016)
6. Karray, A., Feki, M.: Tracking control of a mobile manipulator with fuzzy PD controller. In: 2015 World Congress on Information Technology and Computer Applications (2015)
7. Avanzini, G., Zanchettin, A., Rocco, P.: Constraint-based model predictive control for holonomic mobile manipulators. In: 2015 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS) (2015)
8. Fareh, R., Rabie, T.: Tracking trajectory for nonholonomic mobile manipulator using distributed control strategy. In: 2015 10th International Symposium on Mechatronics and its Applications (ISMA) (2015)
9. Scaglia, G., Rosales, A., Quintero, L., Mut, V., Agarwal, R.: A linear-interpolation-based controller design for trajectory tracking of mobile robots. Control Eng. Pract. **18**(3), 318–329 (2010)
10. Rómoli, S., Serrano, M., Ortiz, O., Vega, J., Eduardo Scaglia, G.: Tracking control of concentration profiles in a fed-batch bioreactor using a linear algebra methodology. ISA Trans. **57**, 162–171 (2015)
11. Gandolfo, D., Rosales, C., Patiño, D., Scaglia, G., Jordan, M.: Trajectory tracking control of a PVTOL aircraft based on linear algebra theory. Asian J. Control. **16**(6), 1849–1858 (2013)
12. Rosales, A., Scaglia, G., Mut, V., di Sciascio, F.: Trajectory tracking of mobile robots in dynamic environments—a linear algebra approach. Robotica **27**(07), 981–997 (2009)
13. Špinka, O., Hanzálek, Z.: Energy-aware navigation and guidance algorithms for unmanned aerial vehicles. In: 2011 IEEE 17th International Conference on Embedded and Real-Time Computing Systems and Applications, vol. 2, pp. 83–88 (2011)
14. Rosales, C., Gandolfo, D., Scaglia, G., Jordan, M., Carelli, R.: Trajectory tracking of a mini four-rotor helicopter in dynamic environments—a linear algebra approach. Robotica **33**(08), 1628–1652 (2014)
15. Andaluz, V., Ortiz, J., Perez, M., Roberti, F., Carelli, R.: Adaptive cooperative control of multi-mobile manipulators. In: IECON 2014—40th Annual Conference of the IEEE Industrial Electronics Society. (2014)
16. Romoli, S., Scaglia, G., Serrano, M., Godoy, S., Ortiz, O., Vega, J.: Control of a fed-batch fermenter based on a linear algebra strategy. IEEE Lat. Am. Trans. **12**(7), 1206–1213 (2014)
17. Andaluz, V., Sásig, E., Chicaiza, W., Velasco, P.: Control Based on Linear Algebra for Mobile Manipulators. Computational Kinematics, pp. 79–86. Springer, Cham (2017)
18. Andaluz, V., Roberti, F., Toibero, J., Carelli, R.: Adaptive unified motion control of mobile manipulators. Control Eng. Pract. **20**(12), 1337–1352 (2012)