



Nonlinear Predictive Control for the Tracking of Unmanned Aerial Vehicles

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Nonlinear Predictive Control for the Tracking of Unmanned Aerial Vehicles

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Abstract—In the following article a nonlinear predictive controller (MPC) is presented as a teaching and learning tool, to test the tracking of different flight paths in a safe way in unmanned aerial vehicles (UAV). This MPC is based on the kinematic model of the UAV and performs the function of minimizing control errors, restricting control actions, increasing system efficiency, maintaining stable flight operation and extending rotor life by restricting UAV input speeds. In addition, the comparison of the data obtained experimentally from Matlab with the data from the DJI Assistant is carried out by simulating the flight path within the virtual environment.

Keywords – virtual environment; nonlinear predictive control; kinematic model; tracking; unmanned aerial vehicles.

I. INTRODUCTION

Robotics is constantly evolving over time and with it comes improved performance in the field of science and technology, giving robots greater autonomy, intelligence and energy efficiency [1]. The services provided by robotics allow to increase productivity, reduce flaws, failures and improve the quality of processes, these services are in great demand not only in the industrial sector, but also in the commercial, educational, medical and domestic sectors, among others [2]. The basic structure that makes up a robot is given by mechanical systems, actuators, sensors and control systems [3]. The types of robots that exist can be classified: (i) according to their function in motion control, autonomous and/or teleoperated; (ii) depending on the function in the environment in which the task is performed, aquatic, terrestrial and aerial [4]. Aerial robots are known as UAVs (unmanned aerial vehicles) [5]. The different applications that UAVs can perform are: (i) search and browse [6]; (ii) security and military applications [7]; (iii) forest fire prevention, mapping and aerial photography [8]; (iv) agriculture and geology [9], among other things. The trend in recent years is to accomplish the task in an efficient and safe way so advanced control algorithms are being developed. It is convenient to test these advanced algorithms in software that allows to safely emulate the operation of UAVs, for which virtual environments are being used [10].

Virtual environments are recreations of real environments and/or environments created on computers that help with the interaction and manipulation of objects. These environments can be oriented to the areas of: (i) teaching - learning; and (ii) training [11]. There are own virtual simulators, such as the DJI Assistant, which allows to calibrate, simulate and obtain the flight information of the UAV. In which you can implement

different advanced control algorithms to analyze the behavior of the system against a specific task.

Advanced control algorithms are automatic control strategies that analyze the behavior of MIMO systems (multiple input - multiple output), time invariant systems, among others [12]. This type of controllers are developed through computer platforms that are in charge of interpreting a mathematical model and evaluating the system's performance through simulations [13]. Among the best known advanced control strategies are: (i) *expert control*, that is the greatest exponent of this type of controller is fuzzy control, which consists of the use of fuzzy algebra in order to represent a resemblance to human thought; (ii) *robust control*, this type of controller defines the characteristics of the system regardless of the disturbances that occur; (iii) *adaptive control*, this type of controller is used in time invariant systems; (iv) *neuronal control*, this type of controller can be compared to the neural networks in the human brain, consisting of a learning stage and a recognition stage; (v) *optimal control*, this type of control is based on the implementation of a functional and an optimization criterion that allows the adjustment of the control objectives; and, (vi) *model based predictive control*, this type of control is based on the future predictions of a system through its past actions [14-15]. However, it should be noted that predictive controllers have been a relevant issue in the field of research and industry at present [16-18].

The idea of model based predictive control (MPC) is that in using an explicit mathematical model, minimizing a target and moving in a sliding horizon [19]. MPC control algorithms are computationally developed to provide a response to a control action [20]. The elements that make up the MPC are: (i) *optimizer*, finds the best result in the performance of a task, also optimizes future control actions; (ii) *cost function*, is a positive function related to an associated cost that varies over the path of the prediction horizon; (iii) *constraints*, are the limits within which the system evolves; and, (iv) *process model*, which describes the behavior of the system and can be linear or nonlinear [21-23]. Ultimately, the advances made in MPC control have positioned it as one of the best controllers when it comes to implementation in kinematic and/or dynamic systems with long sampling periods [24-28].

II. PROBLEM FORMULATION

The proposal of the article is to implement a model based predictive control algorithm (MPC) of an unmanned aerial vehicle (UAV) for autonomous trajectory tracking tasks. So it

was developed mainly in four phases: (i) *modelling*, the non-linear kinematic MIMO model of the UAV will be obtained; (ii) *Control algorithm*, a nonlinear MPC will be implemented, which will take into account the movement restrictions of the UAV, such as the time of computation being less than the sampling period; (iii) *simulation*, a 3D virtual simulation environment shall be developed to analyse the behaviour of the proposed control algorithm when executing autonomous trajectory tracking tasks. In addition, the simulator will allow the provision of external disturbances, e.g. air currents, in order to emulate real flight conditions; (iv) *experimentation*, different experimental tests will be carried out to check the performance of the MPC control algorithm implemented in a Phantom 4 PRO.

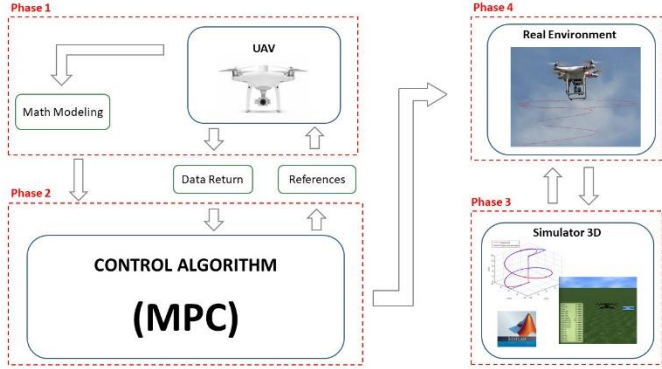


Figure 1. Phases of the proposal.

III. MODELING AND DESIGN OF THE CONTROLLER

In the design of the MPC controller, it should be considered that the optimizer and the cost function play a very important role; the optimizer seeks to minimize the multivariable function and the cost function will correct both the control errors and the control actions to avoid mechanical stress on the UAV motors; furthermore, this controller is directly related to the kinematic model and saturation restrictions of the UAV maneuverability actions.

A. Kinematic Model

The kinematic model of a UAV, represents the displacement in its three dimensions X, Y, Z that found in the inertial reference system $\langle I \rangle$. The kinematic model is obtained by the partial derivation of the point of interest function according to the location of the UAV, $\dot{\mathbf{h}}(t) = \frac{\partial f(\mathbf{q})}{\partial \mathbf{q}} \mathbf{u}(t)$, where $\dot{\mathbf{h}}$ represents the vector of the speeds of the point of interest and being the manoeuvrability vector of the UAV composed of three linear velocities u_l, u_m, u_n with regard to the axes l, m, n of the mobile reference system $\langle S \rangle$; in addition, it has an angular manoeuvring speed in relation to the axis, as shown in Fig. 2.

The movement of the UAV with respect to the inertial reference system is represented as,

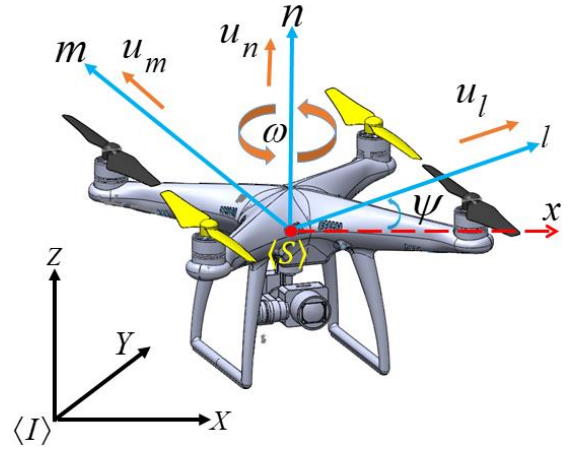


Figure 2. Schematic of the UAV.

so that the movement of the UAV within the inertial plane $\langle I \rangle$ which is represented by the following equation:

$$\begin{cases} \dot{x}_S = u_l \cos(\psi) - u_m \sin(\psi) \\ \dot{y}_S = u_l \sin(\psi) + u_m \cos(\psi) \\ \dot{z}_S = u_n \\ \dot{\psi}_S = \omega \end{cases} \quad (1)$$

and at the same time (1) can be presented in a compact way as,

$$\begin{bmatrix} \dot{x}_S \\ \dot{y}_S \\ \dot{z}_S \\ \dot{\psi}_S \end{bmatrix} = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 & 0 \\ \sin(\psi) & \cos(\psi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_l \\ u_m \\ u_n \\ \omega \end{bmatrix}$$

$$\dot{\mathbf{h}}(t) = \mathbf{J}(\psi) \mathbf{u}(t) \quad (2)$$

where, $\dot{\mathbf{h}}(t) \in \mathfrak{R}^n$ with $n = 4$, which represents the velocity vector on the inertial system $\langle I \rangle$, and the angular velocity with respect to the Z axis; $\mathbf{J}(\psi) \in \mathfrak{R}^{n \times m}$ with $m = n$ is the Jacobian matrix that represents the movement characteristics of the UAV; and $\mathbf{u}(t) \in \mathfrak{R}^n$ defines the manoeuvrability vector of the UAV.

B. MPC Controller

This section describes the implementation of the model-based predictive controller (MPC) with a UAV, the same one that uses directly the mathematical model of the plant by repeatedly solving an optimization problem in a sampling moment. The MPC predicts future system outputs based on current and past system status, its structure show in the following Fig 3,

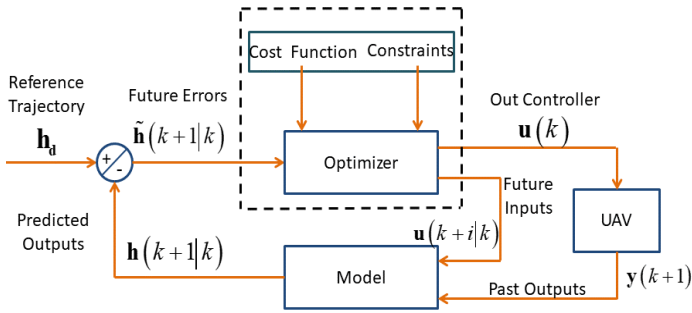


Figure 3. MPC Structure.

given the structure scheme of the MPC, the system model needs to be expressed in a nonlinear state space as shown below:

$$\begin{cases} \dot{\mathbf{h}} = f(\mathbf{h}, \mathbf{u}) \\ \mathbf{y} = g(\mathbf{h}) \end{cases} \quad (3)$$

where, $\mathbf{h} = [x_s \ y_s \ z_s \ \psi_s]$ is the state vector, $\mathbf{u} = [u_{l_s} \ u_{m_s} \ u_{n_s} \ u_{\omega_s}]$ is the vector of the control variables, and is considered as the output vector of the system. Discretizing the nonlinear model, we have:

$$\begin{cases} \mathbf{h}(k+1) = (\mathbf{h}(k), \mathbf{u}(k)) \\ \mathbf{y}(k) = \mathbf{h}(k) \end{cases} \quad (4)$$

where, $\mathbf{y}(k)$ is given by:

$$\mathbf{y}(k) = [x_s(k) \ y_s(k) \ z_s(k) \ \psi_s(k)]^T \quad (5)$$

The predictive control algorithm requires a costing functional F according to the control errors and the variation in the manoeuvrability speeds of the UAV. The cost function will be defined as a quadratic function of the sum of the next errors plus the sum of the predicted increases in the manoeuvrability speeds of the UAV which, based on an optimisation operation, is formulated as follows,

$$F = \sum_{i=1}^N \delta_i \|\tilde{\mathbf{h}}(k+i|k)\|_{\mathbf{Q}}^2 + \sum_{i=1}^N \lambda_i \|\Delta \mathbf{u}(k+i-1|k)\|_{\mathbf{P}}^2 \quad (6)$$

where, k represents the current sampling period, N is the predictive horizon, i is the future prediction, δ_i weighs the efforts of control errors, λ_i weighs up the efforts of the control actions, \mathbf{Q} is the defined positive matrix that weighs the control states, \mathbf{P} is the defined positive matrix that weighs the control actions, i.e., $\mathbf{Q} > 0$ y $\mathbf{P} > 0$.

In addition, $\tilde{\mathbf{h}}(k+i|k)$, represents the future error defined below,

$$\tilde{\mathbf{h}}(k+i|k) = \mathbf{h}(k+i|k) - \mathbf{h}_d(k+i) \quad (7)$$

and $\Delta \mathbf{u}(k+i|k)$, is the variation in control actions that is given by,

$$\Delta \mathbf{u}(k+i|k) = \mathbf{u}(k+i|k) - \mathbf{u}(k+i-1|k) \quad (8)$$

in this way the predictive control algorithm minimizes the target function so that the system is in a desired state given a desired or reference path. Finally, in order to saturate the manoeuvrability speeds $\mathbf{u}(k)$ the maximum and minimum speeds permitted by the UAV are considered to be optimisation constraints, i.e., $\mathbf{u}_{\min} \leq \mathbf{u}(k) \leq \mathbf{u}_{\max}$.

IV. 3D SIMULATION ENVIRONMENT.

A 3D simulator is developed to evaluate the behaviour of the MPC, in the Matlab software, in order to carry out simulation tests prior to the real tests, and so, define the parameters to be implemented experimentally.

In Fig. 4, you can see the interface developed, which allows the entry of the desired path; N ; δ_i ; λ_i and the sample time along with the task execution time; in addition, using the interface options it is possible to simulate the desired flight task of the UAV; display the evolution of control errors; UAV manoeuvrability command and machine run times in each sampling period.

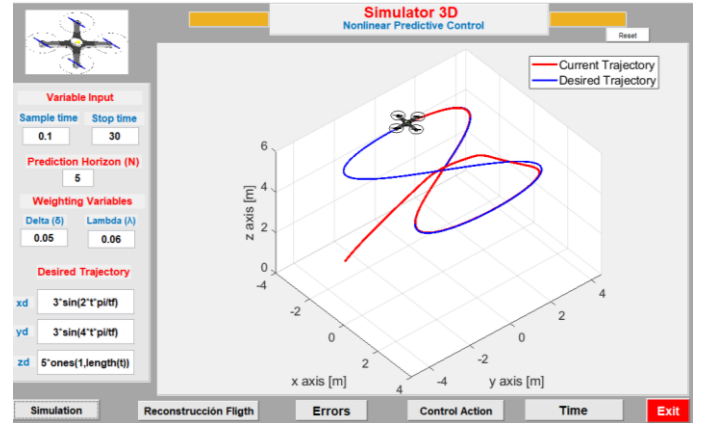


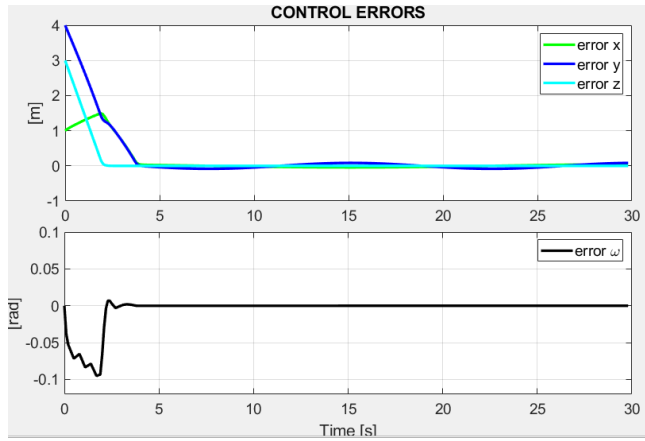
Figure 4. Interface of Simulator 3D of the UAV.

In order to evaluate the performance of the proposed MPC, several tests are executed in the 3D flight simulator developed. Table 1 shows the desired trajectory to be followed and the initial conditions of the UAV. These parameters will be the same for the simulation tests.

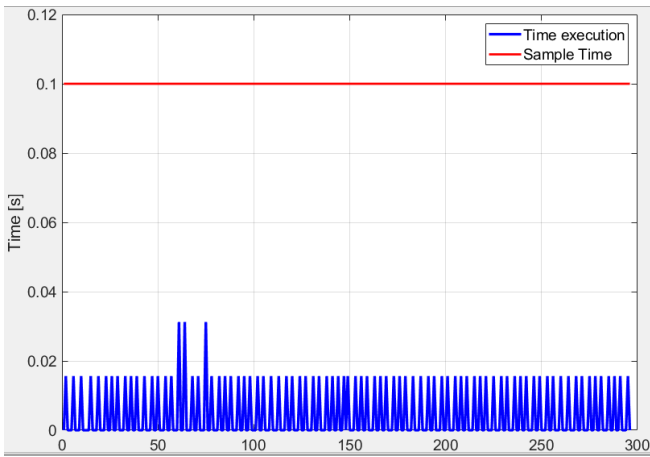
TABLE I. NAVIGATION PARAMETERS CONSIDERING MPC

Initial Parameters	Data	Desired Parameters	Data
x_{o_s}	-1	x_{d_s}	$2 \sin\left(\frac{2\pi}{t} t\right) [m]$
y_{o_s}	-4	y_{d_s}	$\sin\left(\frac{4\pi}{t} t\right) [m]$
z_{o_s}	2	z_{d_s}	$3 [m]$
ψ_{o_s}	$\left(\frac{\pi}{4}\right)$	ψ_{d_s}	$\tan^{-1}\left(\frac{\dot{y}_{d_s}}{\dot{x}_{d_s}}\right) [rad]$

For the first test it is considered that $N = 3$; $\delta_i = 0.6$; $\lambda_i = 0.4$. Fig. 5(a) shows that the control errors obtained given the desired path for the UAV, so that they are close to zero; while Fig. 5(b) shows the computation run time, this is adequate since it is less than the sampling time and therefore has less computational cost.



(a)

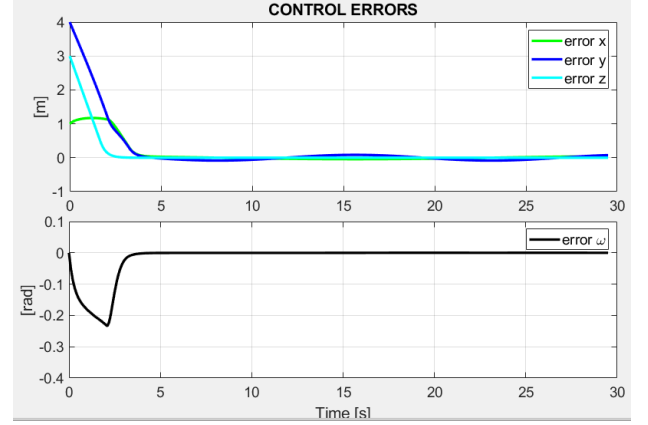


(b)

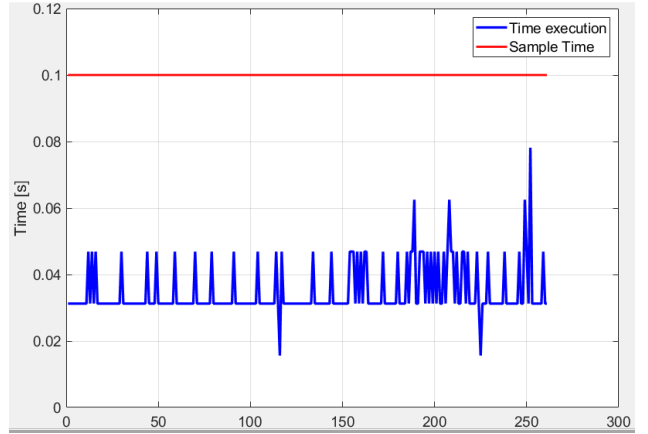
Figure 5. 3D Simulator: (a) control errors, (b) time of execution.

For the second simulation test, it is considered $N = 5$; $\delta_i = 0.6$; $\lambda_i = 0.4$, in such a way that the aim is to enlarge the prediction horizon to analyse the behaviour of the controllability variables.

The Fig. 6(a) shows that the displacement of control errors with respect to x, y, z , it improves remarkably, therefore the control errors are reduced and they do not present oscillations, staying near zero, la Fig. 6(b) shows the effect caused by increasing the prediction horizon as it improves the performance of the controller and allows the appropriate speeds to be obtained for the UAV's flight environment, but it also increases the run time, which can cause problems with the sampling time.



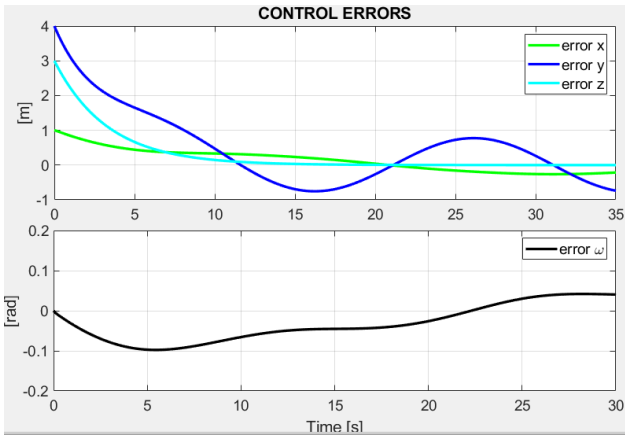
(a)



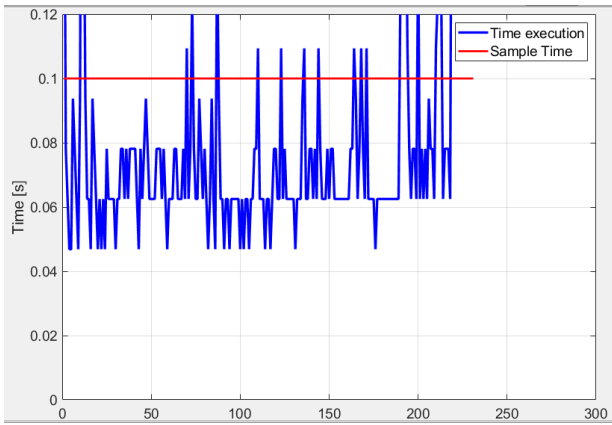
(b)

Figure 6. 3D Simulator: (a) control errors, (b) time of execution.

For the third test it is considered $N = 8$, $\delta_i = 0.6$; $\lambda_i = 0.4$, given the same trajectory as the previous tests. Fig. 7(a) shows how performance increases but compromises computational time in the UAV tracking task, where it can fluctuate and the system takes time to stabilize; while Fig. 7(b) shows that the run time is close to the sampling period where the system becomes unstable because the machine run time is longer than the sampling period.



(a)



(b)

Figure 7. 3D Simulator: (a) control errors, (b) time of execution.

In conclusion, Table 2 shows a detail on the performance of the MPC considering different predictive horizons.

TABLE II. COMPARISON OF CONTROL ERRORS VERSUS VARIATION OF THE PREDICTION HORIZON.

Parameters	$N = 3$	$N = 5$	$N \geq 8$
Control Errors	estable	estable	Inestable
Time of execution	20ms	35ms	$> T_o$

Para todas las simulaciones se considera que $T_o = 100ms$

V. EXPERIMENTAL EVIDENCE

This section presents the experimental tests of the MPC implemented in the Phantom 4 PRO DJI. For the experimental evaluation, the same parameters set out in Table 1 are considered, with a predictive horizon $N=5$ and a $T_o = 100[ms]$. Fig. 8 shows the flight reconstruction of the UAV based on actual navigation data; in the figure it can be seen that the UAV follows the desired trajectory without any problems.

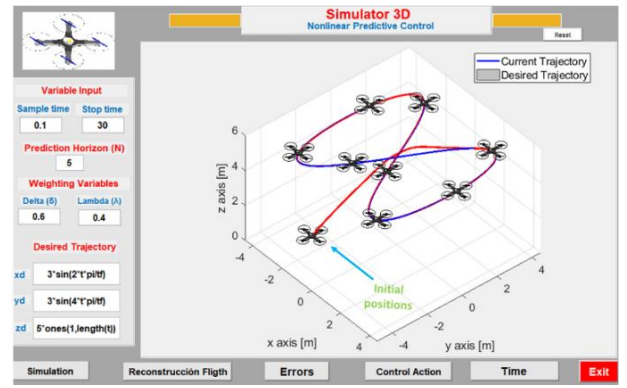


Figure 8. Strobe movement of the UAV, based on real navigation data from the Phantom 4 PRO.

Figure 9 shows the manoeuvrability speeds of the UAV, that do not exceed the maximum and minimum values allowed by the Phantom 4 PRO.

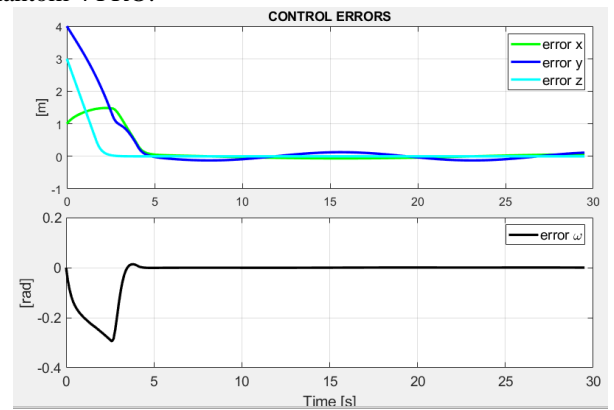


Figure 9. ControlErrors of the UAV.

Finally, Fig. 10 shows that the computed time is less than the sampling time, which ensures that the values obtained in this research are the most suitable for implementation in a real environment.

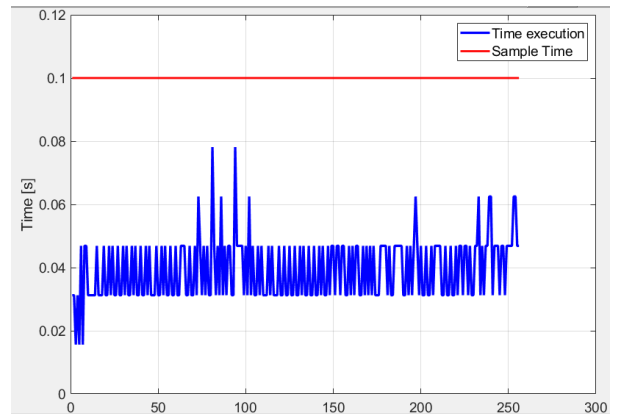


Figure 10. Machine computing time

VI. CONCLUSIONS

In the present work, a nonlinear predictive control algorithm based on an UAV model was designed and implemented, the aim is to carry out trajectory monitoring tasks autonomously.

Additionally, a 3D simulator was developed to evaluate the behavior of the proposed controller. It is important to mention that the prediction horizon must be selected considering that the machine count time is less than the sampling period in order to make the system unstable. Finally, the experimental tests show an adequate functioning of the MPC considering a $T_o = 100[ms]$ with a prediction horizon $N = 5$.

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